Estimated distributions of low flow characteristic – some remarks about their instabilities

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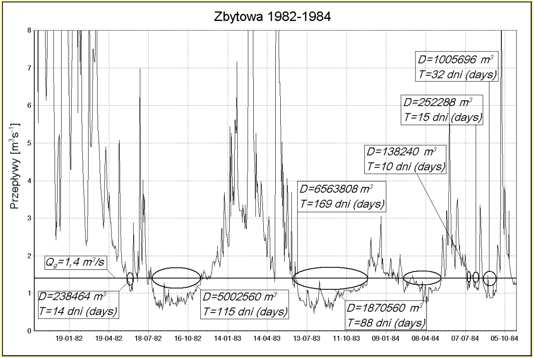
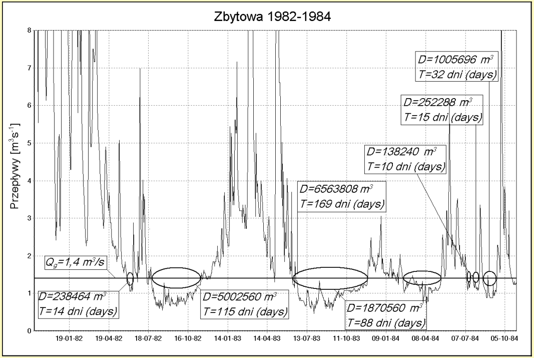
POT method



For given threshold level Q_g define three low flow characteristics:

1. Deficit in [m^3]
$$D_\tau = \int_{t_p}^{t_k} (Q_g - Q(t))\, dt$$
2. Duration [days]
$$T_\tau = t_k - t_p + 1,$$
$$\tau = 1, 2, \ldots$$
3. Minimal outflow P_τ

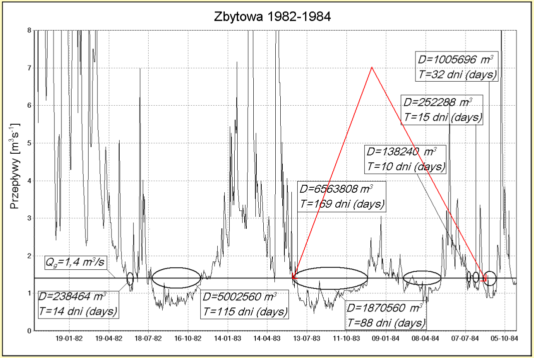
POT method, example



For observed low flows the additional restrictions are imposed [Zelenhaisić and Salvai 1987]:

- Minimum duration m_t;
- Minimum interval between successive low flows m_d;
- Coefficient α setting a minimum deficiency.

POT method, example



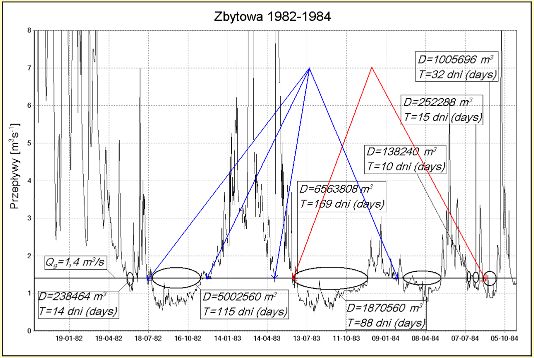
Threshold level $Q_g = Q_{70\%}$

Minimum duration 5 days.

Minimum interval between successive low flows 3 days.

Coefficient: $\alpha = 0,03$.

POT method, example



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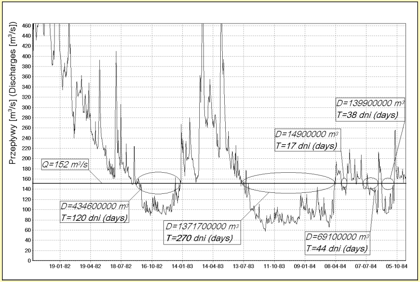
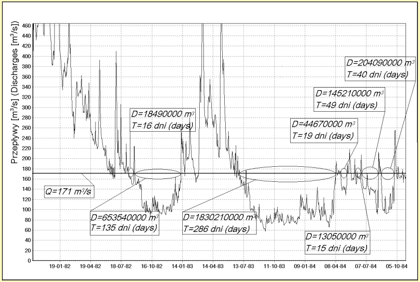
POT method, modification of Q_g

Cigacice 1982 - 84

$Q_{70\%}$



$Q_{60\%}$



$$\Pr(D \leq 1371.7 \text{ millions m}^3 | Q_{70\%}) \leftrightarrow \Pr(D \leq 1830.21 \text{ millions m}^3 | Q_{60\%})$$

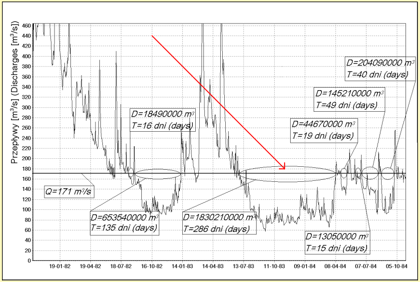
$$\Pr(T \leq 270 \text{ days} | Q_{70\%}) \leftrightarrow \Pr(T \leq 286 \text{ days} | Q_{60\%})$$

POT method, modification of Q_g

Cigacice 1982 - 84

$Q_{70\%}$

$Q_{60\%}$



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The standard approach

Low flow is a period in which the flows are equal to or lower than the accepted threshold Q_g. When determining the flow boundary one can used various criteria. Some of them can be hydrological other economical or ecological. They should depend on the purpose of developing.

Normally:

- Q_g is equal to the highest minimum annual flow;
- $Q_g = Q_{70\%}$ or $Q_g = Q_{90\%}$.

No one looks at the statistical implications of this approach.

Distributions

Assumptions:

- For given Q_g the low flow deficit or duration are having:
  - GEV distribution:

$$F(x) = \exp\left\{ -\left[1 + \xi \left(\frac{x-\mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\}, \; x > \mu - \frac{\sigma}{\xi}$$

  - log-normal distribution:

$$F(x) = \int_0^x \frac{1}{\sqrt{2\pi}\sigma t} \exp\left(-\frac{(\ln t - \mu)^2}{2\sigma^2} \right) dt$$

unknown parameters: μ, σ, ξ or μ, σ are estimated by maximum likelihood method.

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- $Q_{90\%} \leq Q_g \leq Q_{55\%}$ – step Q_g corresponds to the accuracy of flow measurement in the watershed profile.

Tested watershed profiles

Profiles:

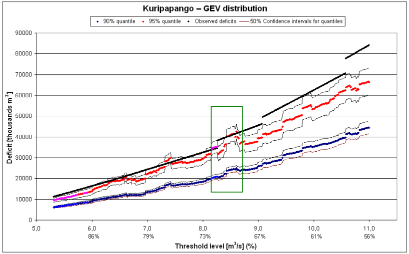
1. Kuripapango (Ngaruroro) (1965-2000)
2. Bogusław (Prosna) (lowland) (1965 - 2005)
3. Bystrzyca Kłodzka (Nysie Kłodzka) (mountains) [1] (1965 - 2005)

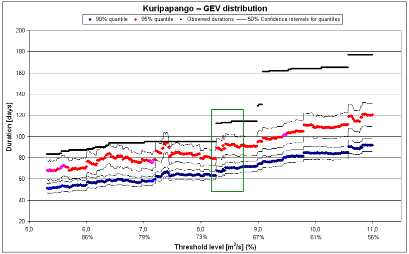
Zelenhaisić and Salvai restrictions:

- Minimum duration $m_t = 5$ days;
- Minimum interval between successive low flows $m_d = 3$ days;
- Setting a minimum deficiency coefficient $\alpha = 0,09$ for Kuropapango and $\alpha = 0,03$ for Bogusław and Bystrzyca Kłodzka.

[1] Polish data were provided by Polish Institue of Meteorology and Water Management

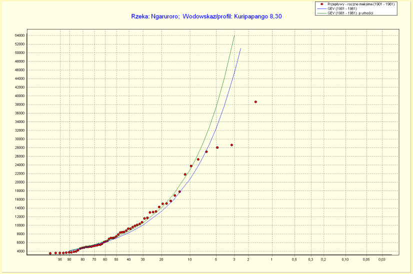
Kuripapango

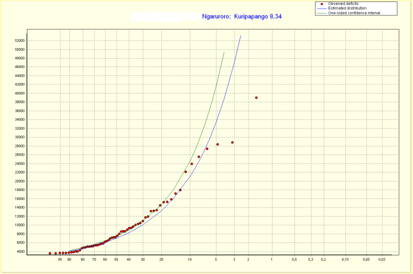




$$Q_{g_1} = 8,30\mathrm{m}^3/\mathrm{s}; \quad Q_{g_2} = 8,34\mathrm{m}^3/\mathrm{s}; \quad \frac{Q_{g_2} - Q_{g_1}}{Q_{g_1}} \approx 0,5\%$$

Kuripapango

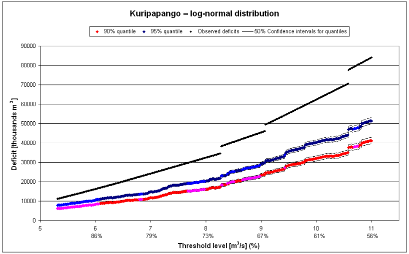


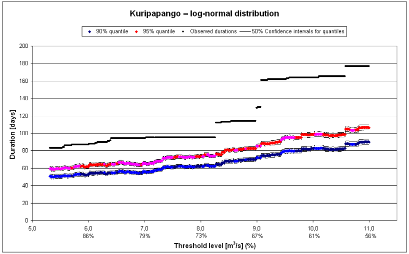


$$p = 0,90 \quad D_{8,30} = 20756; \quad D_{8,34} = 21065 \quad c_1 = 1,5\%$$
$$p = 0,95 \quad D_{8,30} = 32535; \quad D_{8,34} = 33436 \quad c_2 = 2,7\%$$
$$p = 0,99 \quad D_{8,30} = 93171; \quad D_{8,34} = 98942 \quad c_3 = 6,2\%$$

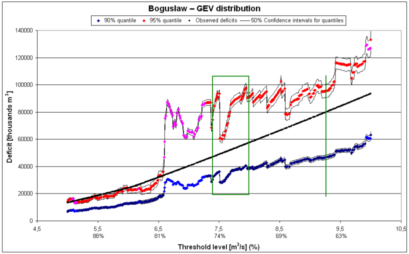
$$D \text{ in thousands m}^3/\text{s}$$

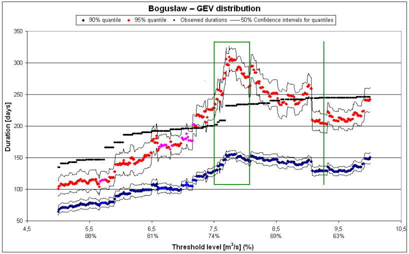
Kuripapango





Bogusław

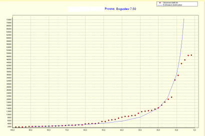


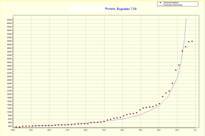


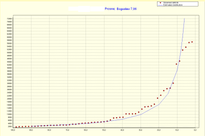
$$Q_{g_1} = 7,94; \quad Q_{g_2} = 7,56; \quad Q_{g_3} = 7,5; \quad Q_{g_4} = 9,25 \mathrm{m}^3/\mathrm{s}$$

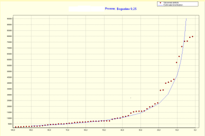
$$d_1 = \frac{Q_{g_2} - Q_{g_1}}{Q_{g_1}} \approx 4,8\% \quad d_2 = \frac{Q_{g_3} - Q_{g_2}}{Q_{g_2}} \approx 0,8\% \quad d_3 = \frac{Q_{g_4} - Q_{g_2}}{Q_{g_2}} \approx 22,4\%$$

Bogusław





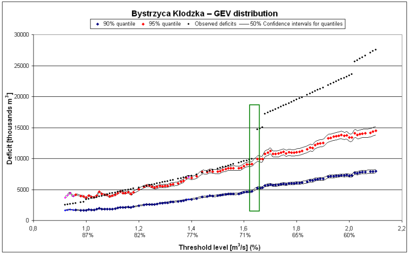


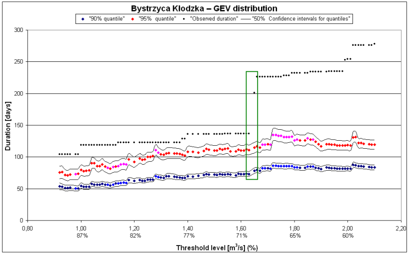


$d_1 = 4,8\%$	$p = 0,90$	$Q_{7,56} = 29096$	$Q_{7,94} = 38529;$	$c_1 = 24,5\%$
	$p = 0,95$	$Q_{7,56} = 62854$	$Q_{7,94} = 90407;$	$c_1 = 30,5\%$
	$p = 0,99$	$Q_{7,56} = 371258$	$Q_{7,94} = 644834;$	$c_1 = 42,4\%$
$d_2 = 0,8\%$	$p = 0,90$	$Q_{7,50} = 36283;$	$Q_{7,56} = 29096$	$c_1 = 24,7\%$
	$p = 0,95$	$Q_{7,50} = 93592;$	$Q_{7,56} = 62854$	$c_1 = 48,9\%$
	$p = 0,99$	$Q_{7,50} = 839337;$	$Q_{7,56} = 371258$	$c_1 = 126,1\%$
$d_3 = 22,4\%$	$p = 0,90$	$Q_{7,56} = 29096;$	$Q_{9,25} = 46671$	$c_1 = 60,4\%$
	$p = 0,95$	$Q_{7,56} = 62854;$	$Q_{9,25} = 95431$	$c_1 = 51,8\%$
	$p = 0,99$	$Q_{7,56} = 371258;$	$Q_{9,25} = 493177$	$c_1 = 32,8\%$

D in thousands m^3/s

Bystrzyca Kłodzka





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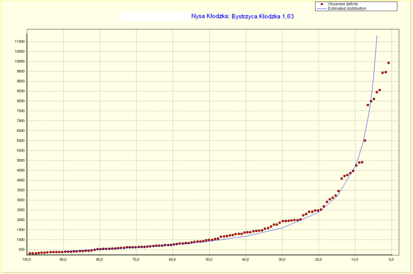

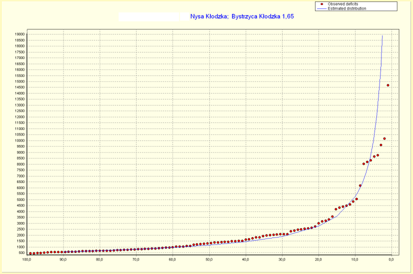

$$Q_{g_1} = 1,65; \quad Q_{g_2} = 1,63$$
$$c_1 = \frac{Q_{g_2} - Q_{g_1}}{Q_{g_1}} \approx 1,2\%$$

$$p = 0,90 \quad c_1 = 11,7\%$$
$$p = 0,95 \quad c_2 = 8,9\%$$
$$p = 0,99 \quad c_3 = 4,3\%$$

Research questions

What are the causes of instability?

Are they the result of numerical, statistical calculations,
or are the property of the catchment?

Thank you for your attention.